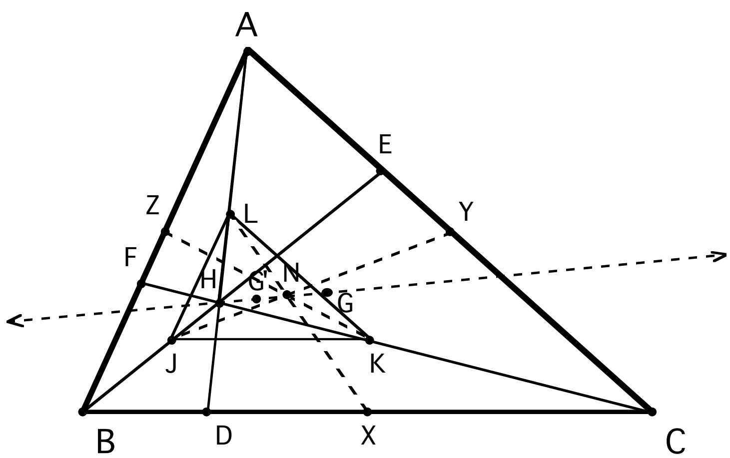
Solution: Concurrency and Euler line locus result

(Refer to the Java sketch at:

http://math.kennesaw.edu/~mdevilli/concurrency-euler-locus.html)

Note that triangle *DEF* (or *D'E'F'*) is homothetic to triangle *ABC*, and therefore homothetic also to the median triangle *KLM*; hence *DK*, *EL* and *FM* are concurrent in the centre of similarity between *DEF* and *KLM*. The result is therefore merely a special case of the Euler line generalization below from De Villiers (2005). Since the locus of *D* is a straight line, it follows from the similarity that the locus of *X* and *X'* is also a straight line, which obviously pass through *O* and *G* (e.g. when *D* respectively coincides with *O* and *A)*. This explains why the locus falls on the Euler line.



Further Euler Line generalization

Given any triangle *ABC* with midpoints of the sides *X*, *Y* and *Z* and three cevians concurrent in *H* as shown. With *H* as centre of similarity and scale factor $\frac{1}{k}$, construct triangle *LJK* similar to *ABC*. Let *N* be the centre of similarity between *LJK* and the median triangle *XYZ*. Then *H*, *N* and *G* are collinear, and $HN = \frac{3}{k-1} NG$. A proof and additional background information can be found in my paper at http://mysite.mweb.co.za/residents/profmd/euler.pdf

Reference

De Villiers, M. (2005). A generalization of the nine-point circle and Euler line. *Pythagoras*, 62, Dec, 31-35.