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A COMPARATIVE STUDY OF TWO VAN HIELE TESTING INSTRUMENTS

Eddie Smith, Bellville College of Education, South Africa

Michael de Villiers, University of Stellenbosch, South Africa

Abstract

This study makes a theoretical and empirical comparison between two Van Hiele testing instruments. These paper-and pencil instruments were developed by researchers at the University of Stellenbosch, Republic of South Africa and Chicago University, U.S.A. respectively. The tests are compared according to the geometric concepts covered, the mathematical processes involved, assignment of students in Van Hiele levels, hierarchical structure and reliability.

Introduction

The Van Hiele model of geometric thought emerged from the companion doctoral dissertations of P M van Hiele and his wife D van Hiele-Geldof. This model posits the existence of five discrete levels of understanding in geometry, each describing characteristics of the thinking process. It is not only descriptive of behaviours of students at certain levels but also predicts other behaviors, e.g. test performance.

In efforts thus far to establish the validity of the Van Hieles' claims, use has been made of clinical interviews (Mayberry, 1981; Burger & Shaughnessy, 1986; Fuys et.al. 1988) and/or paper-and-pencil tests (Usiskin, 1982; De Villiers & Njisaqne, 1987). All these studies verify the general validity and value of the Van Hiele model. In this study two paper-and-pencil test are compared with regards to their descriptive and predictive abilities.

Van Hiele Levels and their characteristics

The 1 through 5 system is used to denote the levels. The index 0 is then reserved for those students who do not operate on Van Hiele's basic level. For brevity, the general characteristic of each level is given.

Level 1:

The student recognizes geometric figures (e.g. square) and relations (e.g. parallel) by their global appearance.

Level 2:

Here the properties of the configuration are perceived. In addition the student acquires the necessary vocabulary to describe the properties.

Level 3:

The student logically orders properties of figures. Class inclusion, definitions and logical implications become meaningful.

Level 4:

The role of deduction is understood.

Level 5:

The student analyzes various deductive systems with a high degree of rigor.

The last level is ignored in this study. This decision can be explained by the fact that it is not covered by South African school syllabi and Usiskin's conclusion that "level 5 either does not exist or is not testable." (Usiskin, 1982: 79).

The major characteristics of the Van Hiele model, as adapted from Usiskin (1982) and Fuys et.al. (1988) are:

- That students move through the levels in a fixed sequence.
- The subject matter that was implicit on level $n - 1$, becomes explicit on level n.
- Each level has its own linguistic symbols and networks connecting these.
- Subject matter is perceived differently at different levels.
- The learning process leading to complete understanding at the next level, has distinct phases. (See Fuys et.al. 1984: 223).

Methodology

Sample

The research group used in the empirical part of the study consisted of 1465 students at 10 Secondary schools in the Western Cape during 1986; 900 were in Grade 9 and 565 were in Grade 10. The choice of schools was arbitrary and depended mainly on the cooperation of the principals: 82.6% of students in Grade 9 were in the 14 – 16 year category and 92.3% of Grade 10 students in the 15 – 17 years group. Female students in Grade 9 were 51.6% as were 35.6% of those in Grade 10.

The Instruments

Test A was developed by the Research Unit for Mathematics Education at the University of Stellenbosch (see University of Stellenbosch, 1984). It consisted of 53 questions, ranging from a simple yes/no response to the construction of formal proofs. The open-ended nature of certain questions, coupled with a level-related scoring scheme made assessment on different levels possible. The following example illustrates this.

Question 31: Is a square a rectangle?
Question 32: Give reasons for your answer in 31.

A response like "31. Yes 32. It looks like a rectangle" would be classified as level 1 as this reasoning reflects visual considerations.
However, a response like "31. Yes 32. A square is a rectangle with equal side" would be classified as level 3 as this entails hierarchical thinking.

Test B was a slightly modified version of the test complied by the Cognitive Development and Achievement in Secondary Geometry Project at Chicago University. The test consisted of 25 multiple choice questions with 8, 7, and 5 questions at levels 1, 2, 3 and 4 respectively. Both tests were applied at every school on consecutive days and the order was varied in which they were written.
A content analysis was done to determine the extent to which geometric

concepts were covered and to ascertain the mathematical processes involved.

Table 1: Comparison of tests according to concepts

		TEST A					**TEST B**		
LEVEL	1	2	3	4		1	2	3	4
Parallel lines	4	6	2	2		1	-	1	1
Triangles	4	1	4	1		3	2	2	2
Quadrilaterals	4	5	11	-		4	5	3	2
Perpendicular lines	-	2	-	1		-	-	-	-
Congruency	-	2	3	1		-	-	-	-

Table 2: Comparison of tests according to processes

		TEST A	**TEST B**
Level 1	Recognition of shapes	7	8
	Creating shapes (drawing)	4	0
	Other e.g. measuring	1	0
Level 2	Identify properties using figures	6	7
	Describing properties	4	0

	Use of appropriate vocabulary	6	0
Level 3	Definitions	5	2
	Logical implication of statement	1	2
	Present short deductive argument	6	0
	Hierarchical classification	8	2
Level 4	Formal proof	5	5

It is apparent from Table 1 that certain concepts are not adequately covered at all levels. In the light of Mayberry's (1981; 89) assertion that students are on different levels for different concepts, it can be claimed that this is a serious limitation in both tests, but generally Test A covers not only more content, but also more processes.

Coding procedure and assignment of students

For each test a "1" was scored if the response to the question met the criterion for that level, and a "0" otherwise. Since the students were mathematically relatively weak, a criterion of 50% for mastery of a level was set. The following procedure was used to place students:

A student was placed on level *n* if he/she matched the criterion for that level as well as for all preceding levels.

A student who did not match the criterion for any of the levels was categorized as level 0.

A student who mastered level *n*, but not all of the preceding levels, was considered unclassifiable with the instrument.

Results

The Van Hiele theory as descriptive model

Analysis of the data was done on a VAX 785 computer using SPSS and SPSSX. The number of students at each Van Hiele level is given in Table 3.

Table 3: Percentages of students at Van Hiele levels

	No level	**1**	**2**	**3**	**4**	**Unclassified**	**Total**
Test A	1.9	22.7	72.1	1.6	0.3	1.4	100
Test B	29.5	40.3	10.2	1.9	0.5	17.5	100

Both tests are useful as testing instruments to assess students' Van Hiele levels, although 82.5% of the students could be classified with Test B as compared the 98.6% with Test A. Despite the high percentages, great differences were observed with respect to individual placement as only 16% of students were placed on exactly the same level. Between 42% and 50.3% showed a shift of one level. These differences in placement are entirely attributed to the mathematical processes involved. For instance, students did better with the drawing of shapes than with the identification of shapes where orientation and shape varied.

Guttman scalogram analysis was used to investigate the hierarchical nature of the levels. Nie et.al. (1975; 532) state that for a truly cumulative and uni-dimensional scale, the reproducibility and scalability coefficients must exceed 0.9 and 0.6 respectively. With Test A both cut-off points were exceeded. With Test B the reproducibility coefficient exceeded 0.9 but the highest value obtained for scalability was 0.519. The hierarchical nature was thus only confirmed by Test A.

Cronbach's alpha coefficient was computed at each level to determine test reliability. For Test A the values ranged from 0.21 to 0.61 and for Test B from

0.05 to 0.54. The higher values for Test A may also be attributed to the larger number of test items.

The Van Hiele theory as predictive model

Here achievement was measured by the student's performance in school in the internal geometry examination of July 1986. A 50% mark was used as criterion and the number of students scoring less than 50% was computed. With both tests it was found that between 70% and 100% of students at levels 0 and 1 failed to match criterion, while this dropped off to between 0% and 20% for students at levels 3 and 4. The data generally confirm the predictive validity of the model with both instruments.

Conclusion

One of the limitations of this study is the homogeneity of the research group with too large a concentration of students at the lower levels. Although Test A outperformed Test B in virtually all aspects, the latter has the advantage of being shorter and more convenient to apply. It is recommended that a standardized paper-and-pencil test be constructed to clinically assess the level at which a student is functioning. In the design the following ought to be considered:

- The geometric concepts to be covered
- The variety of mathematical processes
- Questions at each level for each concept.
- The criterion

Such an instrument would be helpful as a diagnostic instrument for the teacher, as well as for changing the curriculum to adapt to the student's levels of thinking.

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