A dynamic Sketchpad sketch of Cross's Theorem showing measurements of four triangles have the same areas.

Cross's Theorem states that, "Given any triangle ABC with squares constructed on the sides as shown below, then the area of the four shaded triangles are the same. The area of ABC = BEF = ADH =CGI ."

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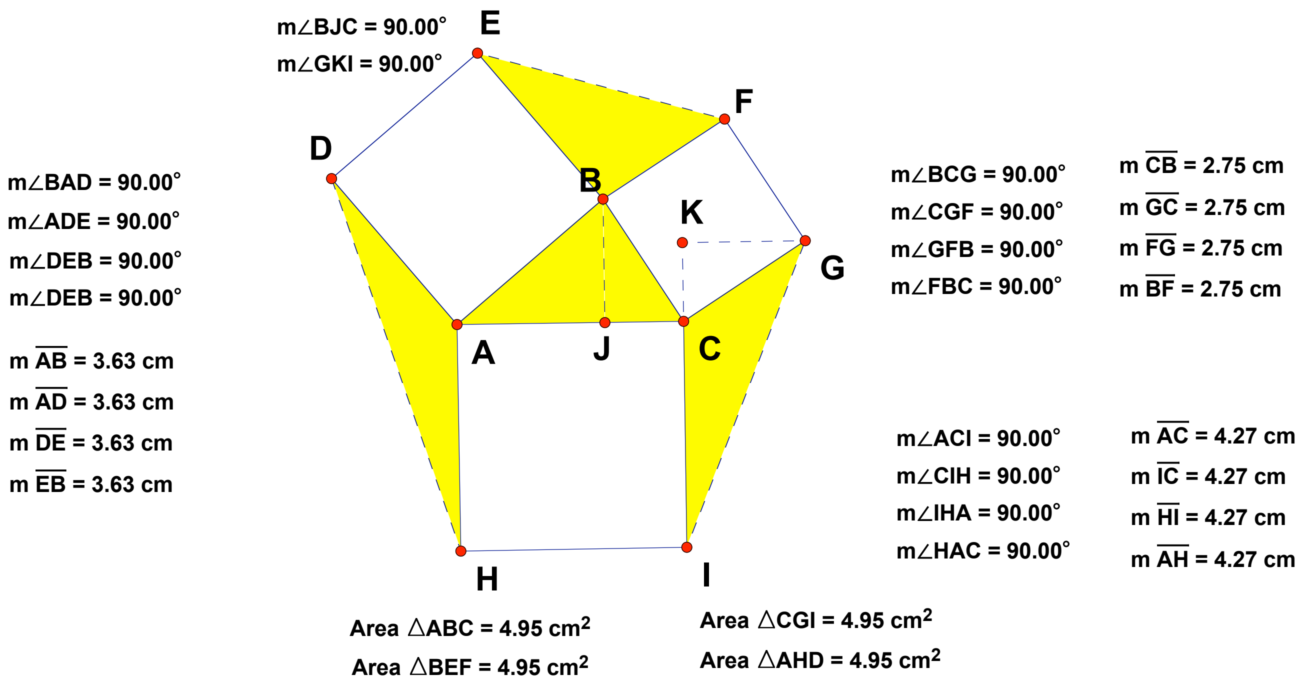

Proof of Cross's Theorem:

m∠BJC = 90.00° m∠BCJ = 57.46° m∠BCK = 32.54° m∠BCJ+m∠BCK = 90.00°

m∠GKI = 90.00° m∠GCK = 57.46° m∠GCK+m∠BCK = 90.00°

The area of the triangle ABC is calculated by the formula 0.5* AC (base of the triangle) * BJ (height of the triangle) and the area of the triangle CGI can be calculated by the same formula 0.5 * CI (base of the triangle) * GK (height of the triangle).

Notice that BJ and GK are obtained by drawing perpendicular line segments to AC and IC, respectively. The point K lies on the ray IC and angle CKG = angle CJB = 90 degrees (by construction). The length of AC = CI (both are sides of the square ACIH by construction as well).

The angle BCJ = GCK (both angles are complementary to angle BCK). The length of BC = CG (both are sides of the square BCGF). The angle CBJ = CGK (both are the same because the other two angles of the triangles are congruent).Thus the triangle CBJ is congruent to the triangle CGK (ASA Postulate). Therefore, the length of BJ = the length of GK (corresponding sides of two congruent triangles), and the area of the triangle ABC = the area of the triangle CGI.

Similarly, we can prove the area of triangle ABC = BEF = ADH = CGI QED